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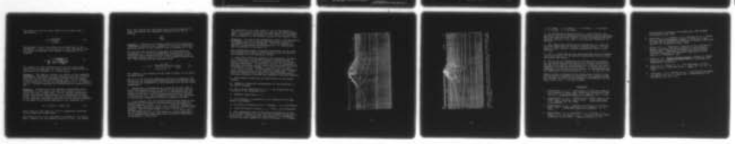

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APPROXIMATE CALCULATION OF NONSTATIONARY TEMPERATURE REGIME OF A ROCK-ICE DAM

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This article is devoted to studies concerning development of principles for calculating nonstationary temperature regimes of rock-ice dams.

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APPROXIMATE CALCULATION OF NONSTATIONARY TEMPERATURE REGIME OF A ROCK-ICE DAM

Engineer N. V. Ukhova

Transactions of Coordinated Meetings on Hydraulic Engineering. Issue No. 23, MISI imeni V. V. Kuybyshev

In order to determine the possibility of building a frozen dam made of local materials under certain harsh climatic conditions, working out its construction in detail, and controlling its thermal regime, it is necessary to be able to predict the temperature field of the dam and foundation. Changes in the temperature of earth dams as a function of the thickness of permafrost soil were investigated by Professor P. A. Bogoslovskiy [1, 2, 3].

In many instances, it is economically advantageous to build a rock-fill dam. In 1961, Gidroproyekt proposed construction of rock-fill dams in regions where permafrost soil occurs, particularly dams with an ice core. The operating regime of such dams differs somewhat from that of earth-fill dams.

During 1961-1964, on order from Gidroproyekt, the Department of Soil Mechanics and Foundations of MISI imeni V. V. Kuybyshev, headed by Professor and Corresponding Member of the Academy of Sciences of the USSR N. A. Tsitovich, studied the principles of calculation and planning of rock-ice dams. The studies took two directions:

1) Development of principles for calculating nonstationary temperature regimes of rock-ice dams;

2) Evaluating the stress-deformation state of the ice core of a rock-ice dam.

Some of the results of these investigations have been published previously [4, 5, 6]. The present article is devoted to the studies which were conducted in the first of the above directions. The studies in the second direction are set forth in another article in the present collection.

As a result of four years' work, an approximate method convenient for practical use was worked out for calculating the nonstationary temperature field of a rock-ice dam and its foundation. The results of the calculations performed by this method for a concrete dam on the Tsypa River were checked on the [illegible]DA 9/60 integrator and by a series of experiments on a

physical model. Comparison of the calculated and experimental temperature fields showed good agreement. This indicated that this calculation method was suitable for practical use.

To develop a method of calculating the nonstationary temperature regime of a rock-ice dam, the following diagram was used (Figure 1).

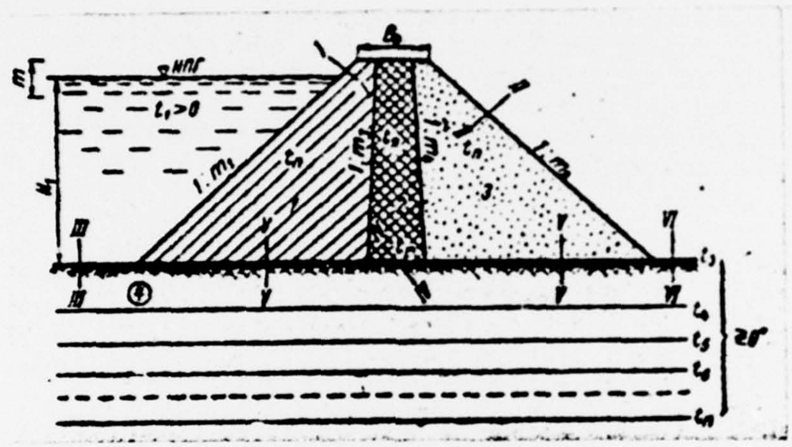


Figure 1. Design Diagram for Rock Dam. I, Rock fill with ice; II, Ice core; III, Dry rock fill; IV, Foundation.

The nonstationary temperature regime of a rock-ice dam and its foundation must be viewed in the general case as a spatial problem. At the present time, this problem cannot be solved in practice. If we exclude from consideration the range in which there is contact between the dam and the edges of the canyon, the problem may be considered a plane one. However, even this problem cannot be solved in a closed form due to its complexity. We must deal with a temperature field which changes as a function of time within the specific, limited, and finite dimensions of an area with different thermal physical characteristics and different initial and boundary conditions in separate sections. Therefore, when working out the method of calculation, the following assumptions were made.

1. The general plane problem is expanded into a number of linear problems and one plane problem, each of which is solved

independently of the others. The general temperature field at any moment in time is composed of partial temperature fields. The linear problems are solved on the basis of classical problems of the theory of thermal conductivity for semi-limited rods with lateral insulation [7]. The plane problem is the distribution of temperature in the foundation, and is solved by the method of finite differences [1].

2. The temperature distribution in the general case is studied by breaking up the nonstationary process into a series of stationary processes, alternating with one another.

3. The construction process is not taken into account. The time is counted from when the dam has already been built and the reservoir is filled. At this moment, certain boundary conditions are established which remain constant during the existence of the structure.

4. The spaces in the fill at the upper wedge are filled with ice. The initial temperature of all the zones in the body of the dam is 0°.

5. In the base of the dam, there is a layer with no cracks, whose cooling and heating take place without any phase shifts.

Then the temperature regime of the rock-ice dam and its foundation (Figure 1) is solved on the basis of the following problems:

1) Thawing of the upper wedge of the dam from the heat in the reservoir, Section I-I;

2) Cooling of parts of the upper and lower wedges and the crest of the dam due to the negative air temperature, Section II-II;

3) Heating of the foundation at a considerable distance from the upper slope by the heat from the reservoir, Section III-III;

4) Cooling of the foundation at a considerable distance from the lower slope due to the negative temperature of the air, Section IV-IV;

5) Thawing, heating or cooling of the cushion of the dam under the influence of the temperature field of the foundation, Section V-V;

6) Thawing, heating, or cooling of the laminar layers (body of the dam - core - foundation), Section VI-VI;

7) Taking into account sets of transformations at the boundary between the frozen and thawed zones;

8) Calculation of forced cooling (cooling galleries);

9) Calculation of the temperature field of the foundation.

The boundary conditions are general for all the problems (Figure 1):

$$\left.\begin{array}{l} t_1(\tau) = \text{const} > 0 \\ t_2(\tau) = \text{const} < 0 \\ t_n(\tau = 0) = t_u(\tau = 0) = 0 \\ t_r(\tau) = \text{const} < 0 \end{array}\right\} \quad (1)$$

where t = f(x) is the temperature of the foundation, x being the depth from the cushion of the dam. In these and subsequent formulas, τ is the calculation time interval and t is the temperature.

In the following we shall examine the solutions of the above problems.

Problem 1. Thawing of the upper wedge due to the heat from the reservoir is accompanied by thawing of the ice in the spaces in the rock fill. Since $t_{\tau=0} = 0°$, when the slope is rather long, the depth of the thawing at a moment in time τ, calculated along a perpendicular to the slope, will be determined by the Stefan formula:

$$\xi = \sqrt{\frac{2\lambda_T t_1 \tau}{\rho \gamma_c w}}, \quad (2)$$

where λ_T is the coefficient of thermal conductivity of the thawed zone;
ρ is the latent heat of melting of the ice;
γ_C is the specific gravity of the skeleton of the rock fill;
w is the moisture content of the rock fill in fractions of a unit.

With a certain granulometric composition for the fill, a considerable influence on the rate of thawing of the upper wedge

may be exerted by convective heat exchange processes. A method was worked out to calculate the influence of convective heat transfer within general heat exchange, with thawing of the upper wedge. This took two forms: the first involved joint solution of heat transfer equations by convection and thermal conductivity as they apply to a concrete scheme, similar to the one used by other researchers [8]. The second involved determining the speed (or flow), and from this the heat flux, with filtration of water in the granular material under the influence of the difference in hydrostatic pressure caused by the different density of the water at different temperatures. Since the heat gradient is very small with natural convection, the determination of analytical relationships is based on Darcy's Law.

Both of these approaches, based upon different original considerations, lead to the same result. Unfortunately, due to the limited scope of this article, we cannot outline in detail the method of calculation of convection and we are forced to present only the final result:[1]

$$\varphi - 1 = 0{,}0006 \frac{\beta (t_2 - t_1) g \gamma^2 n^2 C_p h}{\mu (1-n)^2} d_э^2, \tag{3}$$

where $\phi - \frac{\lambda_э}{\lambda_T}$ is the coefficient of the increase in thermal conductivity due to the presence of convective currents ($\lambda_э$ is the coefficient of the effective thermal conductivity, assuming that all the heat transmitted by contact, in other words by convection and thermal conductivity, is transmitted <u>only</u> by thermal conductivity);

0.0006 is the coefficient which takes into account the hydraulic resistance of the granular layer and the nature of the convective currents in the fill [8];

β is the coefficient of volumetric expansion of water;
t_2 is the temperature of water at maximum density;
t_1 is the temperature of water when frozen;
g is the acceleration due to gravity;
γ is the density of water;
n is the void coefficient of the fill;

---

[1] This question is discussed in detail in [6].

C_p is the specific thermal capacity of the water;

μ is the dynamic coefficient of viscosity;

λ is the coefficient of thermal conductivity of the fill;

$d_э$ is the equivalent diameter of the rock fill, determined by the formula of Ye. A. Zamarin;

h is the depth from the waterline in the reservoir at which the influence of convection is taken into account.

When establishing the constant parameters characterizing the water and the material of the medium (β, t_2, t_1, γ, g, C_p, μ, λ) Formula (3) becomes

$$\varphi - 1 = 42{,}9 \cdot 10^5 \frac{n^3}{(1-n)^3} h d_э^2. \qquad (3')$$

The solution to (3') is in the form of a graph (Figure 2). On this graph, after determining the value of $d_э$ for the fill, with known values for n and h, we can determine $\phi - 1$. There are three areas on the graph: (1) when $\phi - 1 \leq 0.1$ convection has practically no effect; (2) when $0.1 < \phi - 1 \leq 3$ convection affects the positions of the isotherms and must be taken into account; (3) when $3 < \phi - 1$ convection predominates over thermal conductivity.

Depending on the values obtained for $\phi - 1$, convection must be taken into account. When it is necessary to take convection into account, after determining $\phi - 1$ and then using this value to determine $\lambda_э = \phi\lambda_T$, we can substitute $\lambda_э$ in (2) instead of λ_T.

Problem 2. The cooling of portions of the upper and lower wedges and crest of the dam is solved on the basis of the problem of cooling of semi-bounded rods with lateral insulation [7].

The solution has the following form:

$$t(x, \tau) = t_2\left(1 - \operatorname{erf} \frac{x}{2\sqrt{a\tau}}\right), \qquad (4)$$

where (x, τ) is the temperature at a point at a distance x along the normal to the surface, from which cooling is determined at a moment τ;

a is the coefficient of thermal conductivity of the material in the zone in question in a frozen state.

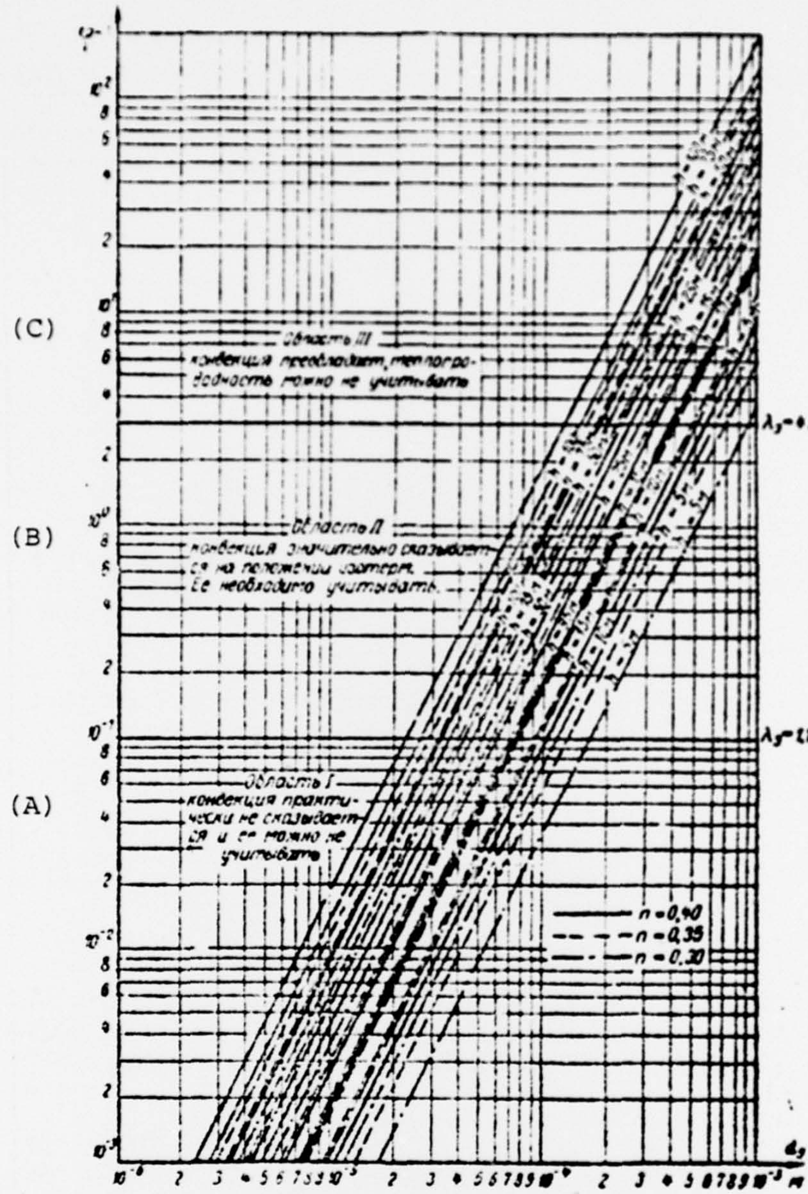


Figure 2. Graph for Determining Convection in General Heat Exchange as a Function of Porosity and Effective Rock Diameter. On the diagram: (A) Region I - convection has practically no effect and may be disregarded. (B) Region II - convection has a considerable influence on the position of the isotherms and must be taken into account. (C) Region III - convection predominates, and thermal conductivity may be disregarded.

Problem 3. Heating of the foundation at a considerable distance from the upper slope on the basis of the problem of Academician A. V. Lykov [7] on the heating of a semi-bounded rod with lateral insulation. The general solution has the form:

$$t(x,\tau) = t_c \operatorname{erfc} \frac{x}{2\sqrt{a\tau}} + \frac{1}{2\sqrt{\pi a\tau}} \int_0^\infty f(\xi) \left[\exp\left(-\frac{(x-\xi)^2}{4a\tau} \right) - \exp\left(-\frac{(x+\xi)^2}{4a\tau} \right) \right] d\xi, \tag{5}$$

where $t(x, \tau)$ is the initial temperature at a point with coordinate x at a moment in time τ;

a is the coefficient of thermal conductivity of the material of the foundation;

$\operatorname{erfc} n = 1 - \operatorname{erf} n$, where $\operatorname{erf} n$ is the integral of the Gaussian errors;

$f(\xi)$ is a function which determines the initial distribution of the temperature in the foundation.

When $f(\xi) = t_i$ = const (1) becomes:

$$t(x,\tau) = (t_1 - t_i) \operatorname{erfc} \frac{x}{2\sqrt{a\tau}} + t_i. \tag{5'}$$

Problem 4. Cooling of the foundation at a considerable distance from the lower slope is solved analogously, by substituting t_1 for t_2 in (5). Otherwise, the method of solving this problem is the same.

Problem 5. Thawing, heating, or cooling of the cushion of the dam under the influence of the temperature field in the foundation must be viewed from two angles;

1) The foundation of the dam is a talik, in other words the rock is at a positive temperature;

2) The base of the dam is made of permafrost soil with a finite thickness (h_m), with the rock beneath at a positive temperature. The solutions to these problems are found on the basis of the heat balance equations proposed by I. S. Moiseyev [5, 6]. In the first case the thawing of the frozen upper wedge on the base side will be expressed by the formula:

$$\Delta\xi_1 = \frac{\lambda_{\tau}(t_i - t_{\kappa})\Delta\tau}{h_i\left[\left(\rho w \gamma_c + \frac{t_{\kappa}\gamma_i}{2}(C_{\kappa} + C_{\text{в}} w)\right)\right]}. \quad (6)$$

Thawing of the ice core on the base side is expressed by the formula:

$$\Delta\xi_2 = \frac{\lambda_{\text{л}}(t_i - t_{\kappa})\Delta\tau}{h_i \rho \gamma_{\text{л}}}. \quad (7)$$

The heating of the dry lower wedge on the base side is described by the formula:

$$\Delta\xi_3 = \frac{\lambda_{\tau}(t_i - t_c)\Delta\tau}{h_i \frac{t_{\kappa}\gamma_i}{2} C_{\kappa}}. \quad (8)$$

In these formulas, λ with the corresponding subscripts represents the coefficients of thermal conductivity of the material in the zones of the dam [in (6), taking convection into account if necessary]; C_K and C_B are the specific heat capacity of the dry fill and water; t_K is the temperature at the contact between the base and dam; h_i is the depth at which the temperature remains constant during the calculation period (according to our calculations on the EGDA 9/60, it may be assumed to be 300-500 meters); ρ is the specific heat of melting of ice and w is the moisture content by weight of the fill.

In the second case the cooling of the frozen upper wedge on the base side is expressed by the formula

$$\Delta\xi_1 = \frac{\lambda_{\text{м}}(-t_{\kappa})\Delta\tau}{h_{\text{м}}\left[\frac{t_{\kappa}\gamma_c}{2}(C_{\kappa} + C_{\text{в}} w)\right]}. \quad (6')$$

The cooling of the ice core from the base side is expressed by:

$$\Delta\xi_2 = \frac{\lambda_{\text{л}}(-t_{\kappa})\Delta\tau}{h_{\text{м}} \frac{t_{\kappa}\gamma_{\text{л}}}{2} C_{\text{л}}}. \quad (7')$$

The cooling of the dry lower wedge from the base side is described by:

$$\Delta\xi_3 = \frac{\lambda_M(-t_x)\Delta\tau}{h_M \frac{t_M\gamma_c}{2} C_M}. \qquad (8')$$

The movement of the lower limit of the permafrost in the foundation ξ_1 during a brief period of time (3-5 years) from the moment the dam goes into service is expressed by the formula:

$$\xi_1 = \sqrt{\frac{0{,}330 a_0 \tau}{0{,}834 - \frac{t_M(h_t - h_M)}{t_t h_M}}}. \qquad (9)$$

The symbols in these formulas are the same as those used before, but the subscript "M" shows that the characteristics were plotted for material in a frozen state.

Problem 6. The thawing, heating, or cooling of the laminated thickness is considered in those cases when the zero isotherm runs from an area with homogeneous thermal physical characteristics into an area with different ones. It is necessary in this connection to use the recommendations of V. S. Lukyanov and M. D. Golovko regarding the average derived characteristics [9].

Problem 7. Calculation of the aggregate conversions takes place when in some region (the core for example) there is a frozen zone and a thawed zone, with an unstable temperature regime and constant temperatures at the boundaries of these zones. Then the shifting of the zero isotherm and the mixing of the remaining isotherms associated with it may be considered on the basis of the known condition at the interface [7]:

$$\Delta\xi = |\lambda_M |\text{grad}\, t_M| - \lambda_T |\text{grad}\, t_T|| \frac{\Delta\tau}{\rho\gamma_\text{л}}, \qquad (10)$$

where $|\text{grad}\ t_M|$ and $|\text{grad}\ t_T|$ are the temperature gradients in the frozen and thawed zones.

The positive value of $\Delta\xi$ indicates an increase in the frozen zone. Formula (10) was compiled for an ice core. For frozen

fill, the term on the right-hand side of this formula which takes into account the aggregate conversions will be as follows:

$$\frac{\Delta \tau}{\rho \gamma_{c} w}.$$

Problem 8. Calculation of forced cooling is considered for a horizontal gallery with a diameter of $2r_0$ with a constantly operating source of cold (runoff) with a thickness of $(-q_0)$.

The solution was obtained on the basis of the problem of Academician A. V. Lykov on temperature distribution in two contiguous semi-limited rods with lateral insulation from a constantly moving source of cold at the boundary between them [7]. The solution has the following form:

$$t(x, \tau) = t_0 + \frac{2q_0 \sqrt{a(\tau_i - \tau_{i-1})}}{\lambda_м \left(\frac{\lambda_т}{\lambda_м} \sqrt{\frac{a_м}{a_т}} + 1 \right)} \mathrm{ierfc} \frac{x}{2\sqrt{a_м \tau}}. \quad (11)$$

The symbols in the formula are the same as those in the formulas given above.

Use of (11) to plot the temperature field in a system as complicated as an inhomogeneous laminated medium (different zones of the dam and foundation) is only possible with the following assumptions:

1. Temperature is determined as a function of time along a radial ray extending from the center of the gallery, and not from its surface; in other words, it is considered that the source of the cold is a point source and the entire gallery is filled with ice. This reduces the effect of the influence of the gallery, producing a reserve of thermal strength.

2. In accordance with the second assumption set forth at the beginning of the article, the calculation is performed for certain moments in time τ_1, τ_2, τ_3, ... from the beginning of reckoning, assuming a steady state condition between these moments. Then for each moment in time, instead of $\tau_i - \tau_{i-1}$, we substitute in (11) $\tau_1 - 0$; $\tau_2 - \tau_1$; $\tau_3 - \tau_2$; etc. The initial temperature t_0 at each moment in time is selected to be an average within the radial ray along which temperature distribution is calculated.

The thickness of the cold source ($- q_0$) is expressed as a function of the method of cooling used and the cooling equipment. Approximate calculation of this factor must be based upon an analysis of the thermal balance of the thawing zones.

Problem 9. In solving the temperature field in the foundation, by contrast with the previous one-dimensional problems, a two-dimensional one is considered. For this purpose, an approach is used which was developed by P. A. Bogoslovskiy and is presented in his paper [1].

The temperature regimes of concrete structures can be calculated conveniently under certain conditions on the basis of the problems set forth above, as follows:

1. Using the structural diagram of the structure, the physical, mechanical, and thermal physical characteristics of various structural zones of the body of the dam and foundation are determined. In individual zones it is necessary to determine only those regions where the characteristics differ considerably from one another: n by 20%; γ by 0.2 m/m^2; a, λ, C by 30 to 50%. In the remaining cases, the average derived characteristics of the zones are established. On the basis of comparison and analysis of these characteristics, the design diagram of the dam and foundation are then prepared.

2. The boundary conditions are established for the design diagram.

a) Geometric dimensions corresponding to the dimensions on the design diagram;

b) The initial conditions $t_n = t_0 = 0$ for temperature distribution in the foundation $f(\xi)$;

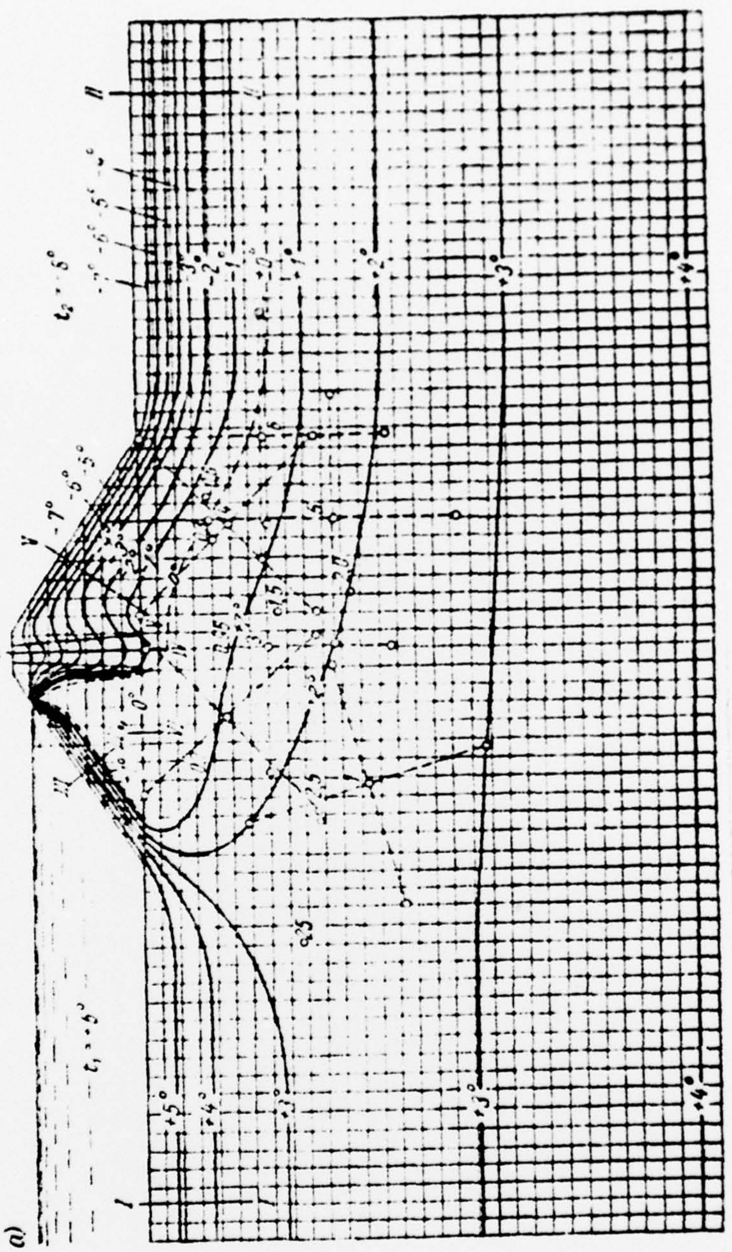
c) Boundary conditions t_1, t_2.

3. The influence of convection on the thawing of the upper wedge is estimated:

a) By using the formula of Ye. A. Zamarin, $d_Э$ is determined;

b) Using the graph in Figure 2, $\phi - 1$ and $\lambda_Э$ are determined.

4. The temperature field of the dam and foundation is built up at certain moments in time from the arbitrary beginning of calculation. For average and large dams, it is recommended that the following moments in time be selected: $\tau_0 = 0$ years,



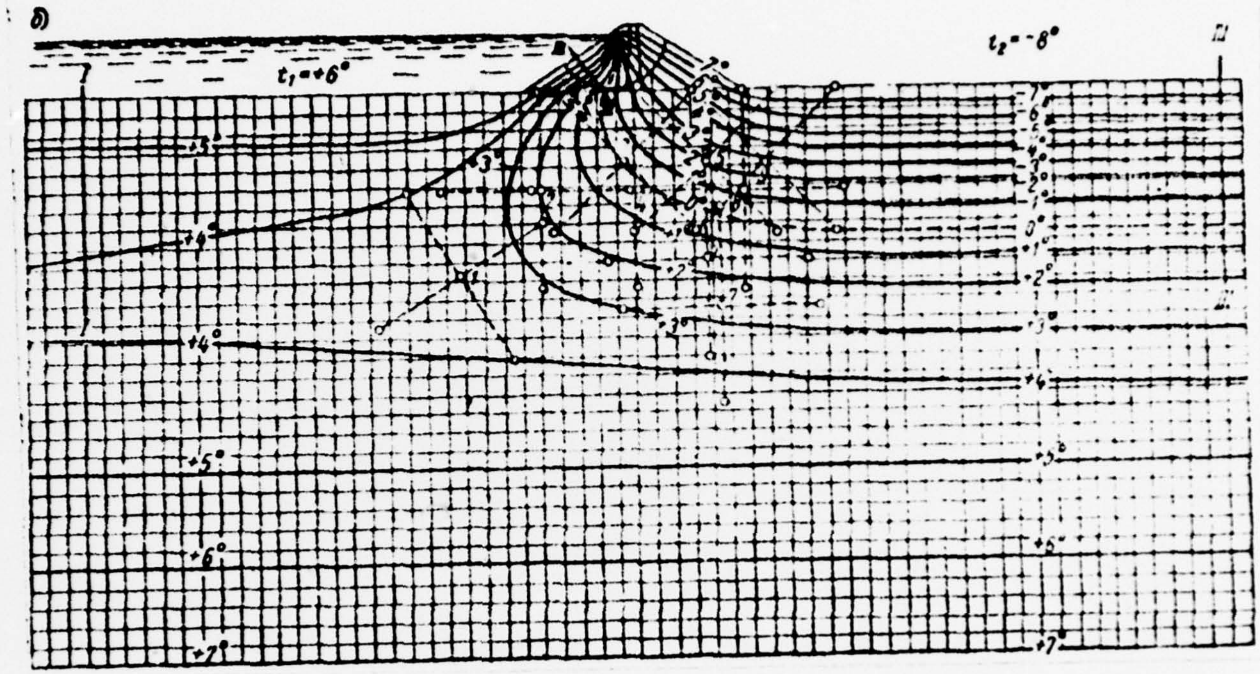


Figure 3. Temperature Fields of Rock-Ice Dam on the Tsypa River and its Foundation, Calculated by the Proposed Method. a: τ = 10 years; b: τ = 75 years.

τ_1 = 5 years; τ_2 = 10 years, τ_3 = 20 years, τ_4 = 40 years; τ_5 = 75 years, τ_6 = 100 years.

5. In the design diagram for each τ_i, cross sections are selected along which temperature calculations may be performed. The cross sections must reflect as fully as possible the temperature changes in the areas of the dam and foundation in question.

6. The temperature values are calculated for τ_i, using the selected cross sections without a cooling gallery. On the basis of these values, a general temperature field is constructed.

7. For τ_2, cross sections are again selected and the temperature and structure of the general temperature field are then calculated once again.

8. As soon as the temperature fields have been calculated for the entire design time period, the thermal stability of the dam and foundation are analyzed and a decision is made regarding the need for forced cooling. If forced cooling is necessary, the influence of the gallery is superimposed upon the temperature field calculated at τ_1, τ_2, τ_i, etc. by (11).

Figure 3 shows an example of the temperature field of a dam and foundation constructed according to the proposed method for the type of rock-ice dam planned by Gidroproyekt for the Tsypa River, with τ = 10 and 75 years.

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